LIBERTY PAPER SET

STD. 12 : Mathematics

Full Solution

Time : 3 Hours **ASSIGNMENT PAPER 10**

PART A

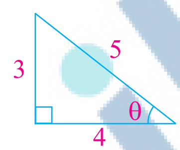
1. (D) **2.** (A) **3.** (A) **4.** (B) **5.** (A) **6.** (D) **7.** (B) **8.** (A) **9.** (A) **10.** (C) **11.** (A) **12.** (A) **13.** (C)
14. (C) **15.** (B) **16.** (C) **17.** (B) **18.** (B) **19.** (C) **20.** (B) **21.** (B) **22.** (A) **23.** (B) **24.** (B) **25.** (C)
26. (A) **27.** (B) **28.** (D) **29.** (C) **30.** (A) **31.** (C) **32.** (C) **33.** (A) **34.** (A) **35.** (A) **36.** (B) **37.** (B)
38. (B) **39.** (B) **40.** (C) **41.** (C) **42.** (A) **43.** (B) **44.** (A) **45.** (C) **46.** (A) **47.** (A) **48.** (B) **49.** (C)
50. (B)

PART B

SECTION A

1.

L.H.S. $= 2\ sin^{-1}\dfrac{3}{5}$



$sin^{-1}\dfrac{3}{5} = \theta$

$\therefore \quad sin\ \theta = \dfrac{3}{5}$

Here, $cos\ \theta = \dfrac{4}{5},\ tan\ \theta = \dfrac{3}{4}$

Now, $2\ sin\ \dfrac{3}{5} = 2\theta$

$$tan\ 2\theta = \frac{2\ tan\ \theta}{1 - tan^2\theta} = \frac{2\left(\frac{3}{4}\right)}{1 - \frac{9}{16}}$$

$$\therefore \quad tan\ 2\theta = \frac{\frac{3}{2}}{\frac{7}{16}}$$

$$\therefore \quad tan\ 2\theta = \frac{24}{7}$$

$$\therefore \quad 2\theta = tan^{-1}\left(\frac{24}{7}\right)$$

$$\therefore \quad 2\ sin^{-1}\frac{3}{5} = tan^{-1}\frac{24}{7}$$

2.

L.H.S. $= cot^{-1}\left[\dfrac{\sqrt{1+sinx}+\sqrt{1-sinx}}{\sqrt{1+sinx}-\sqrt{1-sinx}}\right]$

$$= cot^{-1}\left[\frac{\sqrt{cos^2\frac{x}{2}+sin^2\frac{x}{2}+2sin\frac{x}{2}\cdot cos\frac{x}{2}} + \sqrt{cos^2\frac{x}{2}+sin^2\frac{x}{2}-2sin\frac{x}{2}\cdot cos\frac{x}{2}}}{\sqrt{cos^2\frac{x}{2}+sin^2\frac{x}{2}+2sin\frac{x}{2}\cdot cos\frac{x}{2}} - \sqrt{cos^2\frac{x}{2}+sin^2\frac{x}{2}-2sin\frac{x}{2}\cdot cos\frac{x}{2}}}\right]$$

$$= cot^{-1}\left[\frac{\sqrt{\left(cos\frac{x}{2}+sin\frac{x}{2}\right)^2}+\sqrt{\left(cos\frac{x}{2}-sin\frac{x}{2}\right)^2}}{\sqrt{\left(cos\frac{x}{2}+sin\frac{x}{2}\right)^2}-\sqrt{\left(cos\frac{x}{2}-sin\frac{x}{2}\right)^2}}\right]$$

$$= cot^{-1}\left[\frac{cos\frac{x}{2}+sin\frac{x}{2}+cos\frac{x}{2}-sin\frac{x}{2}}{cos\frac{x}{2}+sin\frac{x}{2}-\left(cos\frac{x}{2}-sin\frac{x}{2}\right)}\right]$$

$$\left\{\begin{aligned} 0 < x < \frac{\pi}{4} &\Rightarrow 0 < \frac{x}{2} < \frac{\pi}{8} \\ &\Rightarrow cos\frac{x}{2} > sin\frac{x}{2} \\ &\Rightarrow cos\frac{x}{2} - sin\frac{x}{2} > 0 \\ &\Rightarrow \left|cos\frac{x}{2} - sin\frac{x}{2}\right| = cos\frac{x}{2} - sin\frac{x}{2} \\ &\Rightarrow \frac{x}{2} \in \left(0, \frac{\pi}{8}\right) \subset (0, \pi) \end{aligned}\right\}$$

$$= cot^{-1}\left[\frac{2cos\frac{x}{2}}{2sin\frac{x}{2}}\right]$$

$$= cot^{-1}\left(cot\ \frac{x}{2}\right)$$

$$= \frac{x}{2} \quad \left(\begin{aligned} &\because\ 0 < x < \tfrac{\pi}{4} \\ &\Rightarrow 0 < \tfrac{x}{2} < \tfrac{\pi}{8} \end{aligned}\right)$$

= R.H.S.

OR

$$\text{L.H.S.} = cot^{-1}\left[\frac{\sqrt{1+sinx}+\sqrt{1-sinx}}{\sqrt{1+sinx}-\sqrt{1-sinx}}\right]$$

$$= cot^{-1}\left[\frac{\sqrt{\frac{1+sinx}{1+sinx}}+\sqrt{\frac{1-sinx}{1+sinx}}}{\sqrt{\frac{1+sinx}{1+sinx}}-\sqrt{\frac{1-sinx}{1+sinx}}}\right]$$

($\because$ Divide numerator and denominator by $\sqrt{1+sinx}$)

$$= cot^{-1}\left[\frac{1+\sqrt{\frac{1-cos\left(\frac{\pi}{2}-x\right)}{1+cos\left(\frac{\pi}{2}-x\right)}}}{1-\sqrt{\frac{1-cos\left(\frac{\pi}{2}-x\right)}{1+cos\left(\frac{\pi}{2}-x\right)}}}\right]$$

$$= cot^{-1}\left[\frac{1+\sqrt{tan^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}}{1-\sqrt{tan^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}}\right]$$

$$= cot^{-1}\left[\frac{1+\left|tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right|}{1-\left|tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right|}\right]$$

$$0 < x < \frac{\pi}{4}$$

$$\therefore\ 0 < \frac{x}{2} < \frac{\pi}{8}$$

$$\therefore\ 0 > -\frac{x}{2} > -\frac{\pi}{8}$$

$$\therefore\ \frac{\pi}{4} > \frac{\pi}{4} - \frac{x}{2} > \frac{\pi}{8}$$

$$\therefore\ tan\left(\frac{\pi}{4}-\frac{x}{2}\right) > 0$$

$$= cot^{-1}\left[\frac{1+tan\left(\frac{\pi}{4}-\frac{x}{2}\right)}{1-tan\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right]$$

$$= tan^{-1}\left[\frac{1-tan\left(\frac{\pi}{4}-\frac{x}{2}\right)}{1+tan\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right]$$

$$= tan^{-1}(1) - tan^{-1}\left(tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)$$

$$= \frac{\pi}{4} - \left(\frac{\pi}{4}-\frac{x}{2}\right) \quad \left(\because \left(\frac{\pi}{4}-\frac{x}{2}\right) \in \left(\frac{\pi}{8}, \frac{\pi}{4}\right) \subset \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right)$$

$$= \frac{x}{2} = \text{R.H.S.}$$

3.

f is continuous at $x = 5$

$$\therefore \lim_{x\to 5^+} f(x) = \lim_{x\to 5^-} f(x) = f(5)$$

$$\therefore \lim_{x\to 5^+} (3x-5) = \lim_{x\to 5^-} (kx+1)$$

$$\left(\begin{array}{l}\because x\to 5^+\\ \Rightarrow x>5\\ \Rightarrow f(x)=3x-5\end{array}\right) \quad \left(\begin{array}{l}\because x\to 5^-\\ \Rightarrow x<5\\ \Rightarrow f(x)=kx+1\end{array}\right)$$

$$\therefore\ 3(5) - 5 = 5k + 1$$

$$\therefore\ 10 = 5k + 1$$

$$\therefore\ 5k = 9$$

$$\therefore\ k = \frac{9}{5}$$

4.

$$I = \int \frac{3x}{1+2x^4}\,dx$$

$$= \int \frac{3x}{(1)^2+\left(\sqrt{2}x^2\right)^2}\,dx$$

Take, $\sqrt{2}\,x^2 = t$

$\therefore\ 2\sqrt{2}\,x\,dx = dt$

$\therefore\ x \cdot dx = \frac{dt}{2\sqrt{2}}$

$$= \int \frac{3}{(1)^2+(t)^2}\,\frac{dt}{2\sqrt{2}}$$

$$= \frac{3}{2\sqrt{2}} \int \frac{dt}{(1)^2+(t)^2}$$

$$= \frac{3}{2\sqrt{2}}\,tan^{-1}(t) + c$$

$$I = \frac{3}{2\sqrt{2}}\,tan^{-1}(\sqrt{2}\,x^2) + c$$

5.

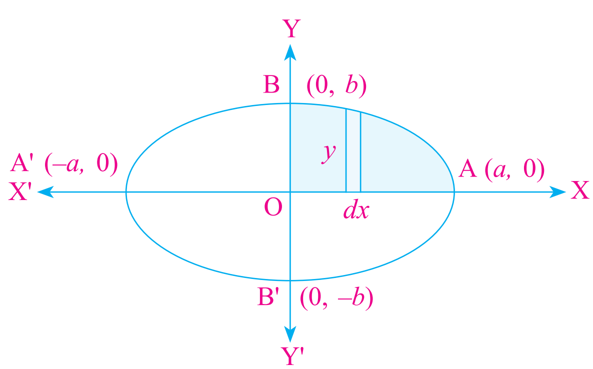
Method 1 :

From Fig, the area of the region ABA'B'A bounded by the ellipse = 4 × area of the region AOBA the first quadrant bounded by the curve x-axis and the ordinates $x = 0$, $x = a$) (as the ellipse is symmetrical about both x-axis and y-axis)

$$= 4\int_0^a y \quad \text{(taking verticalstrips)}$$

Now, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ gives $y = \pm\,\frac{b}{a}\sqrt{a^2-x^2}$

but as the region AOBA lies in the first quadrant, y is taken as positive. So, the required area is,



$$= 4\int_0^a \frac{b}{a}\sqrt{a^2-x^2}\,dx$$

$$= \frac{4b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}sin^{-1}\frac{x}{a}\right]_0^a$$

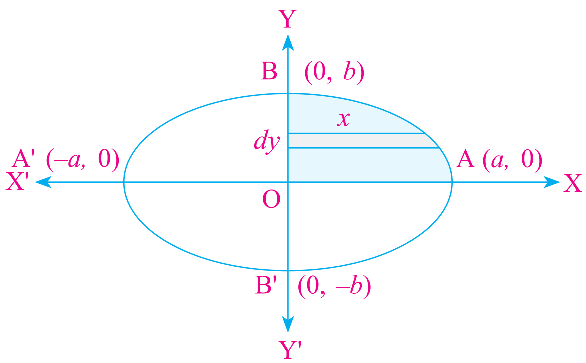
$$= \frac{4b}{a}\left[\left(\frac{a}{2}\times 0+\frac{a^2}{2}sin^{-1}1\right)-(0)\right]$$

$$= \frac{4b}{a}\,\frac{a^2}{2}\,\frac{\pi}{2}$$

$= \pi ab$ sq. unit

Method 2 :

Considering horizontal strips as shown in the Fig, the area of the ellipse is



$$= 4\int_0^b x\,dy$$

$$= \frac{4a}{b}\int_0^b \sqrt{b^2-y^2}\,dy$$

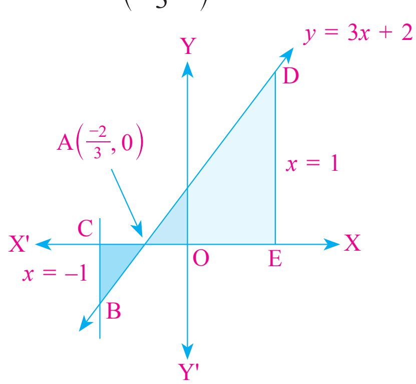
$$= \frac{4a}{b}\left[\frac{y}{2}\sqrt{b^2-y^2}+\frac{b^2}{2}sin^{-1}\frac{y}{b}\right]_0^b$$

$$= \frac{4a}{b}\left[\left(\frac{b}{2}\times 0+\frac{b^2}{2}sin^{-1}(1)\right)-(0)\right]$$

$$= \frac{4a}{b}\,\frac{b^2}{2}\,\frac{\pi}{2} = \pi ab \text{ sq. unit}$$

6.

As shown in the Fig, the line $y = 3x + 2$, meets X-axis at $\left(-\frac{2}{3}, 0\right)$ and its graph lies below X-axis for $x \in \left(-1, -\frac{2}{3}\right)$ and above X-axis for $x \in \left(-\frac{2}{3}, 1\right)$



The required area

= Area of the region ACBA

+ Area of the region ADEA

$$= \left|\int_{-1}^{-\frac{2}{3}}(3x+2)\,dx\right| + \int_{-\frac{2}{3}}^{1}(3x+2)\,dx$$

$$= \left|\left(\frac{3}{2}x^2+2x\right)_{-1}^{-\frac{2}{3}}\right| + \left(\frac{3}{2}x^2+2x\right)_{-\frac{2}{3}}^{1}$$

$$= \left|\left(\frac{3}{2}\left(\frac{4}{9}\right)+2\left(\frac{-2}{3}\right)\right)-\left(\frac{3}{2}(1)+2(-1)\right)\right|$$

$$+ \left(\frac{3}{2}(1)+2(1)\right) - \left(\frac{3}{2}\left(\frac{4}{9}\right)+2\left(-\frac{2}{3}\right)\right)$$

$$= \left|\frac{2}{3}-\frac{4}{3}-\frac{3}{2}+2\right| + \frac{3}{2}+2-\frac{2}{3}+\frac{4}{3}$$

$$= \left|\frac{-1}{6}\right| + \frac{25}{6}$$

$$= \frac{1}{6}+\frac{25}{6}$$

$= \frac{13}{3}$ sq. units

7.

$$\frac{dy}{dx} = \sqrt{4-y^2}$$

$$\therefore \frac{dy}{\sqrt{4-y^2}} = dx$$

→ By integrating both sides,

$$\therefore \int \frac{dy}{\sqrt{4-y^2}} = \int dx$$

$$\therefore sin^{-1}\left(\frac{y}{2}\right) = x + c$$

$$\therefore \frac{y}{2} = sin(x + c)$$

$$\therefore y = 2\,sin(x + c);$$

Which is required general solution of given differential equation.

8.

$$\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$$

$$\vec{c} = 3\hat{i} + \hat{j}$$

$$\vec{a} + \lambda\vec{b} = 2\hat{i} + 2\hat{j} + 3\hat{k} - \lambda\hat{i} + 2\lambda\hat{j} + \lambda\hat{k}$$

$$= (2-\lambda)\hat{i} + (2+2\lambda)\hat{j} + (3+\lambda)\hat{k}$$

$$\to (\vec{a} + \lambda\vec{b}) \perp \vec{c}$$

$$\therefore (\vec{a} + \lambda\vec{b}) \cdot \vec{c} = 0$$

$$\therefore ((2-\lambda)\hat{i} + (2+2\lambda)\hat{j} + (3+\lambda)\hat{k})$$

$$\cdot (3\hat{i} + \hat{j}) = 0$$

$\therefore$ $3(2 - \lambda) + 2 + 2\lambda + 0 = 0$

$\therefore$ $6 - 3\lambda + 2 + 2\lambda = 0$

$\therefore$ $8 - \lambda = 0$

$\therefore$ $\lambda = 8$

9.

A(2, 3, 4), B(–1, –2, 1), C(5, 8, 7)

$\vec{a} = \overrightarrow{AB}$

$= (-1, -2, 1) - (2, 3, 4)$

$\vec{a} = (-3, -5, -3)$

$= -3\hat{i} - 5\hat{j} - 3\hat{k}$

$\vec{b} = \overrightarrow{BC}$

$= (5, 8, 7) - (-1, -2, 1)$

$= 6\hat{i} + 10\hat{j} + 6\hat{k}$

Now, $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ -3 & -5 & -3 \\ 6 & 10 & 6 \end{vmatrix}$

$= 0\hat{i} + 0\hat{j} + 0\hat{k}$

$= \vec{0}$

$\therefore$ A, B and C are collinear

(If $\vec{x} \times \vec{y} = \vec{0}$ then $\vec{x}$ and $\vec{y}$ are colinear)

10.

$\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1} \Rightarrow \frac{x-5}{7} = \frac{y+2}{5} = \frac{z-0}{1}$

L : $\vec{r} = (5\hat{i} - 2\hat{j} + 0\hat{k}) + \lambda(7\hat{i} - 5\hat{j} + \hat{k})$

$\therefore$ $\vec{b_1} = 7\hat{i} - 5\hat{j} + \hat{k}$

$\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$

M : $\vec{r} = (0\hat{i} + 0\hat{j} + 0\hat{k}) + \mu(\hat{i} + 2\hat{j} + 3\hat{k})$

$\therefore$ $\vec{b_2} = \hat{i} + 2\hat{j} + 3\hat{k}$

Now, $\vec{b_1} \cdot \vec{b_2}$

$= (7\hat{i} - 5\hat{j} + \hat{k}) \cdot (\hat{i} + 2\hat{j} + 3\hat{k})$

$= 7 - 10 + 3$

$= 0$

$\therefore$ L and M are perpendicular to each other.

11.

Suppose event E denotes that a randomly selected student studied in class XII and event F denotes that a randomly selected student is girl. We have to find P(E | F).

Now, $P(F) = \frac{430}{1000}$

$= 0.43$

and $P(E \cap F) = \frac{43}{1000}$

$= 0.043$

Therefore, $P(E \mid F) = \frac{P(E \cap F)}{P(F)}$

$= \frac{0.043}{0.43}$

$= 1$

12.

A die is thrown three times, No. of total outcomes.

$n = 216$

Event E : º4 appears on the third loss.

E = {(1, 1, 4), (1, 2, 4), (1, 3, 4), (1, 4, 4), (1, 5, 4), (1, 6, 4), (2, 1, 4), (2, 2, 4), (2, 3, 4), (2, 4, 4), (2, 5, 4), (2, 6, 4), (3, 1, 4), (3, 2, 4), (3, 3, 4), (3, 4, 4), (3, 5, 4), (3, 6, 4), (4, 1, 4), (4, 2, 4), (4, 3, 4), (4, 4, 4), (4, 5, 4), (4, 6, 4), (5, 1, 4), (5, 2, 4), (5, 3, 4), (5, 4, 4), (5, 5, 4), (5, 6, 4), (6, 1, 4), (6, 2, 4), (6, 3, 4), (6, 4, 4), (6, 5, 4), (6, 6, 4)}

$\therefore$ $r = 36$

$\therefore$ $P(E) = \frac{r}{n}$

$= \frac{36}{216}$

$= \frac{1}{6}$

Event F : 6 and 5 appears respectively on first two tosses.

F = {(6, 5, 1), (6, 5, 2), (6, 5, 3), (6, 5, 4), (6, 5, 5), (6, 5, 6)}

$\therefore$ $r = 6$

$\therefore$ $P(F) = \frac{r}{n}$

$= \frac{6}{216}$

$= \frac{1}{36}$

$\therefore$ $E \cap F = \{(6, 5, 4)\}$

$\therefore$ $r = 1$

$\therefore$ $P(E \cap F) = \frac{1}{216}$

$\therefore$ $P(E \mid F) = \frac{P(E \cap F)}{P(F)}$

$= \frac{\frac{1}{216}}{\frac{1}{36}}$

$= \frac{1}{6}$

SECTION B

13.

Here, $f : R \to R$, $f(x) = |x|$

Take, $x_1 = -1$ $\quad f(-1) = |-1| = 1$

Take, $x_2 = 1$ $\quad f(1) = |1| = 1$

$x_1 \neq x_2$ but $f(x_1) = f(x_2)$

$\therefore$ f is not one-one function.

$\forall\ x \in R$, we know that, $|x| \geq 0$

$\therefore f(x) \geq 0$

$\therefore$ Range of $f = [o, \infty) = R^+ \cup \{0\} \neq$ Do-main (R)

$\therefore$ f is not onto function.

14.

$$2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$4X + 6Y = \begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix} \quad \text{........ (1)}$$

($\because$ Multiply by 2)

$$3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$$

Multiply by 3,

$$9X + 6Y = \begin{bmatrix} 6 & -6 \\ -3 & 15 \end{bmatrix} \quad \text{........ (2)}$$

Substract (2) from (1),

$$9X + 6Y = \begin{bmatrix} 6 & -6 \\ -3 & 15 \end{bmatrix}$$

$$4X + 6Y = \begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix}$$

$$- \quad - \quad -$$

$$5X = \begin{bmatrix} 6 & -6 \\ -3 & 15 \end{bmatrix} - \begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix}$$

$$\therefore\ 5X = \begin{bmatrix} 2 & -12 \\ -11 & 15 \end{bmatrix}$$

$$\therefore\ X = \frac{1}{5}\begin{bmatrix} 2 & -12 \\ -11 & 15 \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix} \text{ value}$$

Put, $3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$

$$3\begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix} + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$$

$$\begin{bmatrix} \frac{6}{5} & -\frac{36}{5} \\ -\frac{33}{5} & 9 \end{bmatrix} + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$$

$$\therefore\ 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix} - \begin{bmatrix} \frac{6}{5} & -\frac{36}{5} \\ -\frac{33}{5} & 9 \end{bmatrix}$$

$$\therefore\ 2Y = \begin{bmatrix} 2-\frac{6}{5} & -2+\frac{36}{5} \\ -1+\frac{33}{5} & 5-9 \end{bmatrix} = \begin{bmatrix} \frac{4}{5} & \frac{26}{5} \\ \frac{28}{5} & -4 \end{bmatrix}$$

$$\therefore\ Y = \frac{1}{2}\begin{bmatrix} \frac{4}{5} & \frac{26}{5} \\ \frac{28}{5} & -4 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$$

So, $X = \begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix}$, and $Y = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$

15.

We have $AB = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 5 & -14 \end{bmatrix}$

Since $|AB| = -11 \neq 0$, $(AB)^{-1}$ exists and is given by

$$(AB)^{-1} = \frac{1}{|AB|}\ adj(AB)$$

$$= -\frac{1}{11}\begin{bmatrix} -14 & -5 \\ -5 & -1 \end{bmatrix}$$

$$= \frac{1}{11}\begin{bmatrix} 14 & 5 \\ 5 & 1 \end{bmatrix}$$ {¤u Au.

Further, $|A| = -11 \neq 0$ and $|B| = 1 \neq 0$.

Therefore, A^{-1} and B^{-1} both exist and are given by

and $A^{-1} = -\frac{1}{11}\begin{bmatrix} -4 & -3 \\ -1 & 2 \end{bmatrix}$, $B^{-1} = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$

For $B^{-1}A^{-1} = -\frac{1}{11}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -4 & -3 \\ -1 & 2 \end{bmatrix}$

$$= -\frac{1}{11}\begin{bmatrix} -14 & -5 \\ -5 & -1 \end{bmatrix}$$

$$= \frac{1}{11}\begin{bmatrix} 14 & 5 \\ 5 & 1 \end{bmatrix}$$

So, $(AB)^{-1} = B^{-1}A^{-1}$

16.

Suppose, $u = \left(x + \frac{1}{x}\right)^x$ and $v = x^{\left(1+\frac{1}{x}\right)}$

$\therefore\ y = u + v$

Now, differentiate w.r.t. x,

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \text{......... (1)}$$

Here, $u = \left(x + \frac{1}{x}\right)^x$

Take log both the sides,

$$\log u = x \log\left(x+\frac{1}{x}\right)$$

Now, differentiate w.r.t. x,

$$\therefore \frac{1}{u}\frac{du}{dx} = x\frac{d}{dx}\log\left(x+\frac{1}{x}\right) + \log\left(x+\frac{1}{x}\right)\frac{d}{dx}x$$

$$\therefore \frac{1}{u}\frac{du}{dx} = \frac{x}{\left(x+\frac{1}{x}\right)}\frac{d}{dx}\left(x+\frac{1}{x}\right) + \log\left(x+\frac{1}{x}\right)$$

$$= \frac{x^2}{x^2+1}\left(1-\frac{1}{x^2}\right) + \log\left(x+\frac{1}{x}\right)$$

$$= \frac{x^2}{x^2+1}\left(\frac{x^2-1}{x^2}\right) + \log\left(x+\frac{1}{x}\right)$$

$$\therefore \frac{1}{u}\frac{du}{dx} = \frac{x^2-1}{x^2+1} + \log\left(x+\frac{1}{x}\right)$$

$$\therefore \frac{du}{dx} = u\left(\frac{x^2-1}{x^2+1} + \log\left(x+\frac{1}{x}\right)\right)$$

$$\therefore \frac{du}{dx} = \left(x+\frac{1}{x}\right)^x\left[\frac{x^2-1}{x^2+1} + \log\left(x+\frac{1}{x}\right)\right] \quad \text{... (2)}$$

Now, $v = x^{\left(1+\frac{1}{x}\right)}$

Take log both the sides,

$$\log v = \left(1+\frac{1}{x}\right)\log x$$

Now, differentiate w.r.t. x,

$$\frac{1}{v}\frac{dv}{dx} = \left(1+\frac{1}{x}\right)\frac{d}{dx}\log x + \log x\frac{d}{dx}\left(1+\frac{1}{x}\right)$$

$$\therefore \frac{1}{v}\frac{dv}{dx} = \left(1+\frac{1}{x}\right)\frac{1}{x} + \log x\left(0-\frac{1}{x^2}\right)$$

$$= \frac{x+1}{x^2} - \frac{\log x}{x^2}$$

$$\therefore \frac{dv}{dx} = v\left(\frac{x+1-\log x}{x^2}\right)$$

$$\therefore \frac{dv}{dx} = x^{\left(1+\frac{1}{x}\right)}\left(\frac{x+1-\log x}{x^2}\right) \quad \text{...... (3)}$$

Put the value of equation (2) and (3) in equation (1),

$$\frac{dy}{dx} = \left(x+\frac{1}{x}\right)^x\left[\frac{x^2-1}{x^2+1} + \log\left(x+\frac{1}{x}\right)\right] + x^{\left(1+\frac{1}{x}\right)}\left[\frac{x+1-\log x}{x^2}\right]$$

17.

Here, x and y both are positive.

$x + y = 35 \quad (x < 35,\ y < 35)$

$\therefore x = 35 - y$

$x^2y^5 = (35 - y)^2 y^5$

$\therefore f(y) = (35 - y)^2 y^5$

$\therefore f'(y) = 5y^4 \cdot (35 - y)^2 + y^5 \cdot 2(35 - y)(-1)$

$\therefore f'(y) = 5y^4 \cdot (35 - y)^2 - 2y^5(35 - y)$

$\therefore f'(y) = (35 - y)\, y^4\, (5(35 - y) - 2y)$

$= (35 - y)\, y^4\, (175 - 5y - 2y)$

$= (35 - y)\, y^4\, (175 - 7y)$

$\therefore f'(y) = 7y^4(35 - y)(25 - y)$

$\rightarrow f'(y) = 0$

$\therefore 7y^4(35 - y)(25 - y) = 0$

$\therefore y = 0$ or $35 - y = 0$ fu $25 - y = 0$

$\therefore y = 0$ or $y = 35$ fu $y = 25$

$y = 0, 35$ is not possible. $(\because y \neq 0,\ y < 35)$

$\therefore y = 25$

$y < 25,\ f'(y) > 0$

$y > 25,\ f'(y) < 0$

$\therefore y$ has maximum value $x = 25$

$\therefore$ First number $y = 25$ and

Second number $x = 10$

18.

$\vec{a} = \hat{i} + \hat{j} + \hat{k}$

$\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$

$\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$

$2\vec{a} - \vec{b} + 3\vec{c}$

$= 2\hat{i} + 2\hat{j} + 2\hat{k} - 2\hat{i} + \hat{j} - 3\hat{k} + 3\hat{i} - 6\hat{j} + 3\hat{k}$

$= 3\hat{i} - 3\hat{j} + 2\hat{k}$

Unit parallel vector to the vector $2\vec{a} - \vec{b} + 3\vec{c}$

$$= \frac{2\vec{a} - \vec{b} + 2\vec{c}}{|2\vec{a} - \vec{b} + 3\vec{c}|}$$

$$= \frac{3\hat{i} - 3\hat{j} + 2\hat{k}}{\sqrt{9+9+4}}$$

$$= \frac{3}{\sqrt{22}}\hat{i} - \frac{3}{\sqrt{22}}\hat{j} + \frac{2}{\sqrt{22}}\hat{k}$$

19.

Two lines are parallel

We have $\vec{a_1} = \hat{i} + 2\hat{j} - 4\hat{k}$,

$\vec{a_2} = 3\hat{i} + 3\hat{j} - 5\hat{k}$ and

$\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

Therefore, the distance between the lines is given by

$$d = \left|\frac{\vec{b} \times (\vec{a_2} - \vec{a_1})}{|\vec{b}|}\right|$$

$$= \left| \frac{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 2 & 1 & -1 \end{vmatrix}}{\sqrt{4+9+36}} \right|$$

$$= \frac{|-9\hat{i} + 14\hat{j} - 4\hat{k}|}{\sqrt{49}}$$

$$= \frac{\sqrt{293}}{\sqrt{49}}$$

$$= \frac{\sqrt{293}}{7} \text{ unit}$$

20.

$x \geq 3$

$x + y \geq 5$

$x + 2y \geq 6$

$y \geq 0$

Objective function $Z = -x + 2y$

$x = 3$ ... (i)

$x + y = 5$ ... (ii) $x + 2y = 6$ ... (iii)

x	0	5	
y	5	0	

(0, 5) × (5, 0) ×

x	0	6
y	3	0

(0, 3) × (6, 0) ✓

Solving equation (i) and (ii),

$\therefore y = 5 - 3 = 2$ $\therefore$ (3, 2) ✓

Solving equation (ii) and (iii),

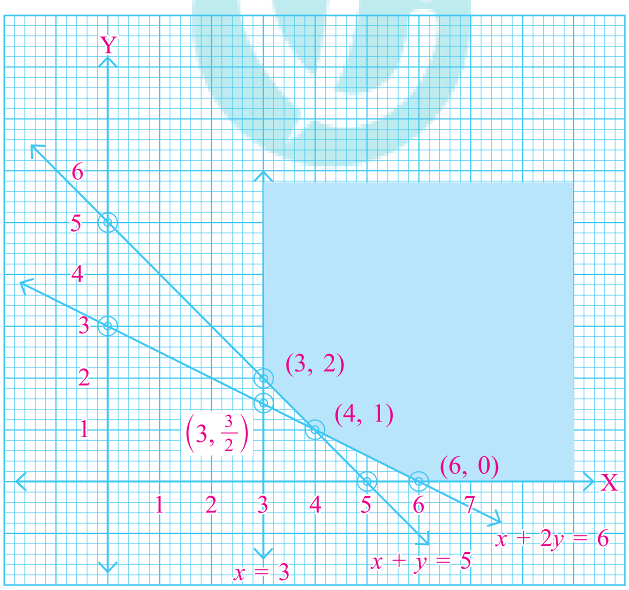
$$\begin{array}{r} x + y = 5 \\ -\, x + 2y = 6 \\ \hline y = 1 \end{array}$$

$\therefore x = 4$ (4, 1) ✓

Solving equation (i) and (iii),

$2y = 3$

$\therefore y = \frac{3}{2}$ $\left(3, \frac{3}{2}\right)$ ×



The shaded region in fig. is feasible region determined by the system of constraints which is unbounded. The corner points of feasible region are (3, 2), (4, 1) abd (6, 0).

Corner Point	Corresponding value of $Z = -x + 2y$
(3, 2)	1 ← Maximum
(4, 1)	–2
(6, 0)	–6

$-x + 2y \leq 1$

Take (6, 4) from unbounded region.

$\therefore -6 + 8 \leq 1$

$\therefore 2 \leq 1$ which is not possible

$\therefore$ The Minimum value of z is not possible.

21.

Event E_1 : First group will win

Event E_2 : Second group will win

Event A : New product introduced was by the second group

$$\therefore P(E_2 \mid A) = \frac{P(E_2) \cdot P(A \mid E_2)}{P(A)}$$

Here, $P(E_1) = 0.6$ and $P(A \mid E_1) = 0.7$

$P(E_2) = 0.4$ and $P(A \mid E_2) = 0.3$

$$\therefore P(A) = P(E_1) \cdot P(A \mid E_1) + P(E_2) \cdot P(A \mid E_2)$$

$$= 0.6 \times 0.7 + 0.4 \times 0.3$$

$$= 0.42 + 0.12$$

$$= 0.54$$

$$\therefore P(E_2 \mid A) = \frac{P(E_2) \cdot P(A \mid E_2)}{P(A)}$$

$$= \frac{0.4 \times 0.3}{0.54}$$

$$= \frac{0.12}{0.54}$$

$$= \frac{12}{54}$$

$$\therefore P(E_2 \mid A) = \frac{2}{9}$$

SECTION C

22.

Here, A and B are symmetric matrices,

$\therefore$ A' = A and B' = B ... (1)

Take, X $= AB - BA$

X' $= (AB - BA)'$

$= (AB)' - (BA)'$

$= B'A' - (A'B')$

$= BA - AB$ $\quad (\because$ Result (1))

$= -(AB - BA)$

$= -X$

$\therefore$ X is skew symmetric matrix.

$\therefore$ AB – BA is skew symmetric matrix.

23.

$$\begin{bmatrix}1 & -1 & 2\\0 & 2 & -3\\3 & -2 & 4\end{bmatrix}\begin{bmatrix}-2 & 0 & 1\\9 & 2 & -3\\6 & 1 & -2\end{bmatrix}$$

$$= \begin{bmatrix}-2-9+12 & 0-2+2 & 1+3-4\\0+18-18 & 0+4-3 & 0-6+6\\-6-18+24 & 0-4+4 & 3+6-8\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

So, $\begin{bmatrix}1 & -1 & 2\\0 & 2 & -3\\3 & -2 & 4\end{bmatrix}^{-1} = \begin{bmatrix}-2 & 0 & 1\\9 & 2 & -3\\6 & 1 & -2\end{bmatrix}$

Now, given system of equations can be written, in matrix form as follows :

$AX = B$

$$\begin{bmatrix}1 & -1 & 2\\0 & 2 & -3\\3 & -2 & 4\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}1\\1\\2\end{bmatrix}$$

OR $$\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}1 & -1 & 2\\0 & 2 & -3\\3 & -2 & 4\end{bmatrix}^{-1}\begin{bmatrix}1\\1\\2\end{bmatrix}$$

$$= \begin{bmatrix}-2 & 0 & 1\\9 & 2 & -3\\6 & 1 & -2\end{bmatrix}\begin{bmatrix}1\\1\\2\end{bmatrix}$$

$$= \begin{bmatrix}-2+0+2\\9+2-6\\6+1-4\end{bmatrix} = \begin{bmatrix}0\\5\\3\end{bmatrix}$$

Therefore, $x = 0$, $y = 5$ and $z = 3$.

24.

Method 1 :

$y = sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-\left(\frac{2x}{1+x^2}\right)^2}} \cdot \frac{d}{dx}\left(\frac{2x}{1+x^2}\right)$$

$$= \frac{1}{\sqrt{1-\frac{4x^2}{(1+x^2)^2}}} \times \frac{(1+x^2)\frac{d}{dx}2x - 2x\frac{d}{dx}(1+x^2)}{(1+x^2)^2}$$

$$= \frac{(1+x^2)}{\sqrt{(1+x^2)^2-4x^2}} \times \frac{(1+x^2)(2)-(2x)(2x)}{(1+x^2)^2}$$

$$= \frac{1}{\sqrt{(1+x^2)^2-4x^2}} \times \frac{2+2x^2-4x^2}{(1+x^2)}$$

$$= \frac{1}{\sqrt{1+2x^2+x^4-4x^2}} \times \frac{2-2x^2}{1+x^2}$$

$$= \frac{2(1-x^2)}{\sqrt{1-2x^2+x^4}} \times \frac{1}{1+x^2}$$

$$= \frac{2(1-x^2)}{\sqrt{(1-x^2)^2}} \times \frac{1}{1+x^2}$$

$$= \frac{2(1-x^2)}{|1-x^2|} \times \frac{1}{1+x^2} \qquad \text{............ (1)}$$

Option 1 : $|x| < 1$

$\therefore -1 < x < 1$

$\therefore 0 < x^2 < 1$

$\therefore 0 < 1 - x^2$

$\therefore |1 - x^2| = 1 - x^2$

$$\frac{dy}{dx} = \frac{2(1-x^2)}{(1-x^2)} \times \frac{1}{1+x^2}$$

$$\therefore \frac{dy}{dx} = \frac{2}{1+x^2}$$

Option 2 : $|x| > 1$

$\therefore x < -1$ and $x > 1$

$\therefore x^2 > 1$

$\therefore 1 - x^2 < 0$

$\therefore |1 - x^2| = -(1 - x^2)$

$$\frac{dy}{dx} = \frac{2(1-x^2)}{-(1-x^2)} \times \frac{1}{x^2-1}$$

$$\therefore \frac{dy}{dx} = \frac{-2}{1+x^2}$$

Option 3 : Take, $x = \pm 1$

$\therefore \frac{dy}{dx}$ does not exist.

Method 2 :

$y = sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Suppose, $x = tan\,\theta \qquad \theta \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

$\theta = tan^{-1}x$

Now, $y = sin^{-1}\left(\frac{2tan\theta}{1+tan^2\theta}\right)$

$$y = sin^{-1}(sin2\theta) \qquad \text{............ (1)}$$

Option 1 : $-1 < x < 1$

$$\tan\left(\frac{-\pi}{4}\right) < \tan\theta < \tan\frac{\pi}{4}$$

$$\frac{-\pi}{4} < \theta < \frac{\pi}{4}$$

$$\frac{-\pi}{2} < 2\theta < \frac{\pi}{2}$$

$$2\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \subset \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \quad \dots (1)$$

$\therefore\ y = \sin^{-1}(\sin 2\theta)$

$= 2\theta \quad (\because$ From equation (1))

$\therefore\ y = 2\tan^{-1}x$

$$\frac{dy}{dx} = 2 \cdot \frac{d}{dx}\tan^{-1}x$$

$$\therefore\ \frac{dy}{dx} = \frac{2}{1+x^2}$$

Option 2 : $x > 1$

$$\therefore\ \tan\theta > \tan\frac{\pi}{4}$$

$$\therefore\ \frac{\pi}{4} < \theta < \frac{\pi}{2}$$

$$\therefore\ \frac{\pi}{2} < 2\theta < \pi$$

$$\therefore\ \frac{\pi}{2} - \pi < 2\theta - \pi < 0$$

$$\therefore\ \frac{-\pi}{2} < 2\theta - \pi < 0$$

$$(2\theta - \pi) \in \left(-\frac{\pi}{2}, 0\right) \subset \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \quad \dots (2)$$

Now, $-\sin(2\theta - \pi)$

$= \sin(\pi - 2\theta)$

$= \sin 2\theta$

$\therefore y = \sin^{-1}(\sin 2\theta)$

$= \sin^{-1}(-\sin(2\theta - \pi))$

$= -\sin^{-1}(\sin(2\theta - \pi))$

$= -(2\theta - \pi) \quad (\because$ From equation (2))

$\therefore y = \pi - 2\theta$

$= \pi - 2\tan^{-1}x$

$= \pi - 2\tan^{-1}x$

Differentiate w.r.t. x,

$$\frac{dy}{dx} = -2\frac{d}{dx}\tan^{-1}x$$

$$\therefore\ \frac{dy}{dx} = \frac{-2}{1+x^2}$$

Option 3 : $x < -1$

$-\infty < x < -1$

$$\therefore\ \tan\left(\frac{-\pi}{2}\right) < \tan\theta < \tan\left(\frac{-\pi}{4}\right)$$

$$\therefore\ \frac{-\pi}{2} < \theta < \frac{-\pi}{4}$$

$$\therefore\ -\pi < 2\theta < \frac{-\pi}{2}$$

$$\therefore\ 0 < 2\theta + \pi < \frac{-\pi}{2} + \pi$$

$$\therefore\ 0 < 2\theta + \pi < \frac{\pi}{2}$$

$$(2\theta + \pi) \in \left(0, \frac{\pi}{2}\right) \subset \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \quad \dots (3)$$

Now, $-\sin(2\theta + \pi)$

$= \sin 2\theta$

$y = \sin^{-1}(\sin 2\theta)$

$= \sin^{-1}(-\sin(2\theta + \pi))$

$= -\sin^{-1}(\sin(2\theta + \pi))$

$= -(2\theta + \pi) \quad (\because$ From equation (3))

$= -2\theta - \pi$

$y = -2\tan^{-1}x - \pi$

$$\therefore\ \frac{dy}{dx} = \frac{-2}{1+x^2}$$

Option 4 : Take, $x = \pm 1$

$\therefore\ \frac{dy}{dx}$ does not exist.

OR

Method 3 :

$$y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$= 2\tan^{-1}x,\ \forall\ x \in R$

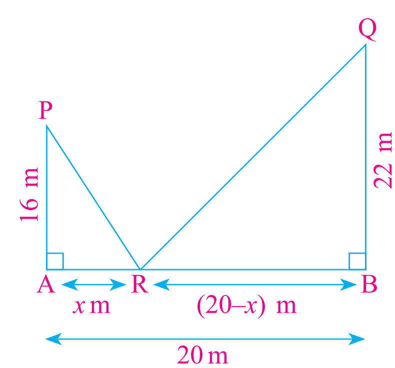
$$\therefore\ \frac{dy}{dx} = 2 \cdot \frac{d}{dx}\tan^{-1}x = \frac{2}{1+x^2}$$

25.

Let R be a point on AB such that

AR = x m

$\therefore$ RB = $(20 - x)$ m $(\because$ AB = 20 m)



From figure, $RP^2 = AR^2 + AP^2$

and $RQ^2 = RB^2 + BQ^2$

$\therefore\ RP^2 + RQ^2 = AR^2 + AP^2 + RB^2 + BQ^2$

$= x^2 + (16)^2 + (20 - x)^2 + (22)^2$

$= 2x^2 - 40x + 1140$

Suppose, $S \equiv S(x)$

$= RP^2 + RQ^2$

$= 2x^2 - 40x + 1140$

$\therefore \quad S'(x) = 4x - 40$

Now, Take $S'(x) = 0$, we get $x = 10$

Also, $S''(x) = 4 > 0, \forall x$

and therefore, $S''(10) > 0$

Therefore, by second derivative test, $x = 10$ is the point of local minima of S. Thus, the distance of R from A on AB $= x = 10$ m.

26.

$$I = \int tan^{-1}\sqrt{\frac{1-x}{1+x}}\, dx$$

Take, $x = cos\,\theta$,

$\therefore\ dx = -sin\,\theta\, d\theta$

$$I = \int tan^{-1}\sqrt{\frac{1-cos\,\theta}{1+cos\,\theta}}\,(-sin\,\theta)\, d\theta$$

$$I = -\int tan^{-1}\sqrt{tan^2\frac{\theta}{2}}\, sin\,\theta\, d\theta$$

$$= -\int tan^{-1}\left(tan\,\frac{\theta}{2}\right) sin\,\theta\, d\theta$$

$$= -\int \frac{\theta}{2}\cdot sin\,\theta\, d\theta$$

$$I = \frac{-1}{2}\int \theta\cdot sin\,\theta\, d\theta$$

$$I = \frac{-1}{2}\, I_1 \qquad \text{... (1)}$$

$$I_1 = \int \theta \cdot sin\,\theta\, d\theta$$

$\rightarrow$ Now, $u = \theta$; $v = sin\,\theta$

$\frac{du}{d\theta} = 1$

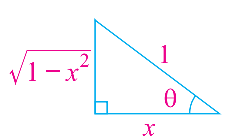
Using integration by parts rule,

$$I_1 = \theta\int sin\,\theta\, d\theta - \int\left(1\int sin\,\theta\, d\theta\right) d\theta$$

$$= -\theta\cdot cos\,\theta + \int cos\,\theta\, d\theta$$

$$I_1 = -\theta\, cos\,\theta + sin\,\theta + c$$

$\rightarrow$ Now, $x = cos\,\theta$



$\theta = cos^{-1}\, x$

$\sqrt{1-x^2} = sin\,\theta$

$$I_1 = -cos^{-1}x\cdot x + \sqrt{1-x^2} + c_1$$

$\rightarrow$ Put the value of I_1 in equation (1),

$$I = \frac{-1}{2}\left[-x\cdot cos^{-1}x + \sqrt{1-x^2} + c_1\right] + c'$$

$$I = \frac{1}{2}\left[x\cdot cos^{-1}x - \sqrt{1-x^2}\right] + c$$

$$\left(\because\ \frac{-1}{2}c_1 + c' = c\right)$$

27.

Method 1 :

$(x^3 - 3xy^2)\, dx = (y^3 - 3x^2y)\, dy$

$$\therefore\ \frac{dy}{dx} = \frac{x^3 - 3xy^2}{y^3 - 3x^2y}$$

$$\therefore\ \frac{dy}{dx} = \frac{1 - 3\left(\frac{y}{x}\right)^2}{\left(\frac{y}{x}\right)^3 - 3\left(\frac{y}{x}\right)} \qquad \text{... (1)}$$

Take, $\frac{y}{x} = v$

$\therefore\ y = vx$

$\rightarrow$ Differentiate w.r.t. x,

$$\therefore\ \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$\rightarrow$ Put this value of equation (1),

$$\therefore\ v + x\frac{dv}{dx} = \frac{1-3v^2}{v^3-3v}$$

$$\therefore\ x\frac{dv}{dx} = \frac{1-3v^2}{v^3-3v} - v$$

$$\therefore\ x\frac{dv}{dx} = \frac{1-3v^2-v^4+3v^2}{v^3-3v}$$

$$\therefore\ x\frac{dv}{dx} = \frac{1-v^4}{v^3-3v}$$

$$\therefore\ \left(\frac{v^3-3v}{1-v^4}\right) dv = \frac{dx}{x}$$

$\rightarrow$ Take integration both the side,

$$\therefore\ \int\frac{v^3}{1-v^4}\, dv - 3\int\frac{v\, dv}{1-v^4} = \int\frac{dx}{x}$$

$$\therefore\ -\frac{1}{4}\int\frac{-4v^3}{1-v^4}\, dv + 3\int\frac{v}{v^4-1}\, dv = \int\frac{dx}{x}$$

$$\therefore\ -\frac{1}{4}\int\frac{-4v^3}{(1-v^4)}\, dv + 3\int\frac{v\ dv}{(v^2)^2-1} = \int\frac{dx}{x}$$

$\rightarrow$ In second term of integration $v^2 = t$,

$\therefore\ 2v\, dv = dt$

$$\therefore\ v\, dv = \frac{dt}{2}$$

$$\therefore -\frac{1}{4}\int \frac{\frac{d}{dv}(1-v^4)}{(1-v^4)}\, dv + \frac{3}{2}\int \frac{dt}{t^2-1} = \int \frac{dx}{x}$$

$$\therefore -\frac{1}{4} \log|1-v^4| + \frac{3}{2}\cdot\frac{1}{2} \log\left|\frac{t-1}{t+1}\right| = \log|x| + \log|c'|$$

$$\therefore -\frac{1}{4} \log|1-v^4| + \frac{3}{4} \log\left|\frac{v^2-1}{v^2+1}\right| = \log|xc'|$$

$$\therefore \frac{1}{4} \log\left|\frac{1}{1-v^4}\right| + \frac{3}{4} \log\left|\frac{v^2-1}{v^2+1}\right| = \log|c'x|$$

$$\therefore \log\left|\left(\frac{1}{1-v^4}\right)^{\frac{1}{4}}\right| + \log\left|\left(\frac{v^2-1}{v^2+1}\right)^{\frac{3}{4}}\right| = \log|c'x|$$

$$\therefore \log\left|\frac{1}{(1-v^4)^{\frac{1}{4}}} \times \frac{(v^2-1)^{\frac{3}{4}}}{(v^2+1)^{\frac{3}{4}}}\right| = \log|c'x|$$

$$\therefore \frac{(v^2-1)^{\frac{3}{4}}}{(v^4-1)^{\frac{1}{4}}} \times \frac{1}{(v^2+1)^{\frac{3}{4}}} = c'x$$

$$\therefore \frac{(v^2-1)^{\frac{3}{4}}}{(v^2-1)^{\frac{1}{4}}(v^2+1)^{\frac{1}{4}}(v^2+1)^{\frac{3}{4}}} = c'x$$

$$\therefore \frac{(v^2-1)^{\frac{1}{2}}}{v^2+1} = c'x$$

$$\rightarrow v = \frac{y}{x}$$

$$\therefore \frac{\left[\left(\frac{y}{x}\right)^2-1\right]^{\frac{1}{2}}}{\left(\frac{y}{x}\right)^2+1} = c'x$$

$$\therefore \frac{\left[y^2-x^2\right]^{\frac{1}{2}}}{x} \times \frac{x^2}{y^2+x^2} = c'x$$

$$\therefore \left(y^2-x^2\right)^{\frac{1}{2}} = c'(x^2+y^2)$$

$$\therefore (y^2-x^2) = (c')^2[x^2+y^2]^2$$

$$\therefore (x^2-y^2) = -(c')^2[x^2+y^2]^2$$

$$\therefore (x^2-y^2) = c[x^2+y^2] \quad (\because -(c')^2 = c)$$

Method 2 :

$x^2 - y^2 = c(x^2 + y^2)$

$$\therefore \frac{x^2-y^2}{\left(x^2+y^2\right)^2} = c$$

Differentiate w.r.t. x,

$$\frac{d}{dx}\left[\frac{x^2-y^2}{\left(x^2+y^2\right)^2}\right] = 0$$

$$\therefore (x^2+y^2)^2\left(2x-2y\frac{dy}{dx}\right) - (x^2-y^2)\cdot 2(x^2+y^2)\left(2x+2y\frac{dy}{dx}\right) = 0$$

$$\therefore 2(x^2+y^2)\left[\left(x^2+y^2\right)\left(x-y\frac{dy}{dx}\right) - \left(2x^2-2y^2\right)\left(x+y\frac{dy}{dx}\right)\right] = 0$$

$$\therefore x^3 - x^2y\frac{dy}{dx} + xy^2 - y^3\frac{dy}{dx} - 2x^3 - 2x^2y\cdot\frac{dy}{dx} + 2xy^2 + 2y^3\frac{dy}{dx} = 0$$

$$\therefore y^3\frac{dy}{dx} - 3x^2y\frac{dy}{dx} - x^3 + 3xy^2 = 0$$

$$\therefore \frac{dy}{dx}(y^3 - 3x^2y) = x^3 - 3xy^2$$

$$\therefore (y^3 - 3x^2y)\, dy = (x^3 - 3xy^2)\, dx$$

$$\therefore (x^3 - 3xy^2)\, dy - (y^3 - 3x^2y)\, dy = 0$$

Which is general solution of given differential.